JEPA as Categorical World Model

Joint Embedding Predictive Architecture via Coalgebras

FunctorFlow.jl

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Introduction

JEPA (Joint Embedding Predictive Architecture) is LeCun's proposed framework for building world models that predict in *representation space* rather than *observation space*. The key insight is that predicting pixel-by-pixel is intractable, but predicting *embeddings* of future states is feasible.

FunctorFlow.jl reveals that JEPA is fundamentally a coalgebraic construction:

JEPA Component	Categorical Analog
Encoder	Coalgebra morphism
Predictor	Endofunctor dynamics
Target encoder (EMA)	Frozen reference coalgebra
Prediction loss	Obstruction to commutativity
World model	F-coalgebra (X □ F(X))

JEPA Component	Categorical Analog
Optimal representation	Final coalgebra (Lambek's lemma)

```
using FunctorFlow
```

Part 1: Coalgebra — World Models as State Transitions

A coalgebra is a pair (X, α : X → F(X)): a state space equipped with a transition structure. This is the categorical formalization of a world model.

```
# Build a simple world model diagram
D = world_model_block(;
    name=:SimpleWorldModel,
    observation_object=:Pixels,
    latent_object=:Embedding,
    encoder_name=:encode,
    dynamics_name=:predict_next,
    decoder_name=:decode,
)

println("Objects: ", join(keys(D.objects), ", "))
println("Operations: ", join(keys(D.operations), ", "))
println()

# The coalgebra structure
coalgebras = get_coalgebras(D)
for (name, c) in coalgebras
    println(c)
end
```

```
Objects: Pixels, Embedding
Operations: encode, predict_next, decode, encode_then_predict, autoencoder

Coalgebra(:coalgebra, Embedding →_{identity} Embedding)
```

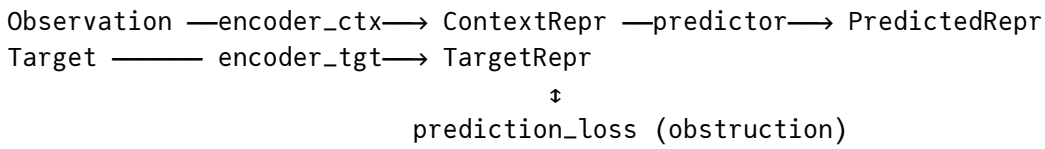
The coalgebra declares that `predict_next` is the world model dynamics: given a latent state, it produces the next latent state.

```
# Bind concrete implementations
bind_morphism!(D, :encode, x -> x ./ 255.0)           # normalize
bind_morphism!(D, :predict_next, x -> x .+ 0.01)      # simple drift
bind_morphism!(D, :decode, x -> x .* 255.0)            # denormalize

compiled = compile_to_callable(D)
result = FunctorFlow.run(compiled, Dict(
    :Pixels => [128.0, 64.0, 200.0],
))

println("Encoded:  ", round.(result.values[:encode]; digits=4))
println("Predicted:", round.(result.values[:predict_next]; digits=4))
println("Decoded:  ", round.(result.values[:autoencoder]; digits=4))
```

```
Encoded:  [0.502, 0.251, 0.7843]
Predicted:[0.512, 0.261, 0.7943]
Decoded:  [128.0, 64.0, 200.0]
```

Part 2: JEPA Block — Prediction in Embedding Space

The JEPA block encodes the fundamental prediction-in-embedding pattern as a FunctorFlow diagram:




```
# Build a JEPA block
D_jepa = jepa_block(;
    name=:ImageJEPA,
    observation_object=:Context,
    target_object=:MaskedRegion,
    context_repr=:CtxEmb,
    target_repr=:TgtEmb,
    comparator=:l2,
)

println("=== JEPA Diagram: $(D_jepa.name) ===")
println("\nObjects:")
```

```
for (name, obj) in D_jepa.objects
    println("  $(name) :: $(obj.kind)")
end
println("\nMorphisms:")
for (name, op) in D_jepa.operations
    if op isa Morphism
        role = get(op.metadata, :role, :unknown)
        net = get(op.metadata, :network, :none)
        detach = get(op.metadata, :detach, false)
        println("  $(name): $(op.source) → $(op.target) [role=$(role), net=$(net), detach=$(d
    elseif op isa Composition
        println("  $(name): $(op.source) → $(op.target) [chain=$(op.chain)]")
    end
end
println("\nLosses:")
for (name, loss) in D_jepa.losses
    println("  $(name): paths=$(loss.paths), comparator=$(loss.comparator)")
end
println("\nCoalgebra:")
for (name, c) in get_coalgebras(D_jepa)
    println("  ", c)
end
```

```
=== JEPA Diagram: ImageJEPA ===

Objects:
  Context :: observation
  MaskedRegion :: observation
  CtxEmb :: representation
  TgtEmb :: representation

Morphisms:
  context_encoder: Context -> CtxEmb [role=encoder, net=online, detach=false]
  target_encoder: MaskedRegion -> TgtEmb [role=encoder, net=target, detach=true]
  predictor: CtxEmb -> TgtEmb [role=predictor, net=none, detach=false]
  prediction_path: Context → TgtEmb [chain=[:context_encoder, :predictor]]

Losses:
  prediction_loss: paths=[(:prediction_path, :target_encoder)], comparator=l2

Coalgebra:
  Coalgebra(:jepa_dynamics, CtxEmb →_{identity} CtxEmb)
```

Part 3: JEPA Execution — The Obstruction Loss

The prediction loss IS an obstruction loss: it measures how far the encoder-predictor square is from commuting.

```
# Bind implementations: encoders as linear scaling
bind_morphism!(D_jepa, :context_encoder, x -> x .* 0.5)
bind_morphism!(D_jepa, :target_encoder, x -> x .* 0.5)  # same encoder (EMA would make it clos
bind_morphism!(D_jepa, :predictor, identity)  # trivial predictor

compiled_jepa = compile_to_callable(D_jepa)

# Case 1: Context and target are the same → perfect prediction
result1 = FunctorFlow.run(compiled_jepa, Dict(
    :Context => [1.0, 2.0, 3.0],
    :MaskedRegion => [1.0, 2.0, 3.0],  # same input
))
println("Case 1 (same input): loss = ", round(result1.losses[:prediction_loss]; digits=6))
println("  Predicted: ", result1.values[:prediction_path])
println("  Target:    ", result1.values[:target_encoder])

# Case 2: Different context and target → non-zero loss
result2 = FunctorFlow.run(compiled_jepa, Dict(
    :Context => [1.0, 2.0, 3.0],
    :MaskedRegion => [4.0, 5.0, 6.0],  # different!
))
println("\nCase 2 (different input): loss = ", round(result2.losses[:prediction_loss]; digits=
println("  Predicted: ", result2.values[:prediction_path])
println("  Target:    ", result2.values[:target_encoder])
```

```
Case 1 (same input): loss = 0.0
  Predicted: [0.5, 1.0, 1.5]
  Target:    [0.5, 1.0, 1.5]

Case 2 (different input): loss = 2.598076
  Predicted: [0.5, 1.0, 1.5]
  Target:    [2.0, 2.5, 3.0]
```

When the loss is zero, the JEPA square commutes — the encoder is an exact coalgebra morphism.

Part 4: KAN-JEPA — Merging KET and JEPA

KAN-JEPA uses a left Kan extension (Σ) as the predictor, merging the KET attention pattern with JEPA's embedding-space prediction:

```
D_kan = kan_jepa_block(;
    name=:KanJEPA_Demo,
    observation_object=:Tokens,
    target_object=:MaskedTokens,
    context_repr=:TokenEmb,
    target_repr=:MaskedEmb,
    relation_object=:Neighborhood,
    reducer=:mean,
)

println("=== KAN-JEPA Diagram ===")
for (name, op) in D_kan.operations
    if op isa KanExtension
        println("  Σ($(op.source); along=$(op.along)) → $(op.target)  [$(op.reducer)]")
    elseif op isa Morphism
        role = get(op.metadata, :role, :unknown)
        println("  $(name): $(op.source) → $(op.target) [$(role)]")
    end
end
println("\nThis is the natural fusion:")
println("  KET's attention = left-Kan aggregation (Σ)")
println("  JEPA's loss = obstruction to commutativity")
```

```
=== KAN-JEPA Diagram ===
  context_encoder: Tokens → TokenEmb [encoder]
  target_encoder: MaskedTokens → MaskedEmb [encoder]
  Σ(TokenEmb; along=Neighborhood) → MaskedEmb  [mean]

This is the natural fusion:
  KET's attention = left-Kan aggregation (Σ)
  JEPA's loss = obstruction to commutativity
```

Part 5: Hierarchical JEPA (H-JEPA)

H-JEPA uses multiple abstraction levels for multi-scale prediction. Fine levels handle short-range details; coarse levels handle long-range abstract planning.

```
D_hjepa = hjepa_block(;
    name=:MultiScaleJEPA,
    levels=[:fine, :medium, :coarse],
)

println("=== H-JEPA: 3 Abstraction Levels ===")
println("\nObjects per level:")
for level in [:fine, :medium, :coarse]
    obs = Symbol("$(level)__Obs_$(level)")
    repr = Symbol("$(level)__CtxRepr_$(level)")
    println("  $(level): $(obs) → $(repr)")
end

println("\nAbstraction morphisms:")
for (name, op) in D_hjepa.operations
    if op isa Morphism && get(op.metadata, :role, nothing) == :abstraction
        from = op.metadata[:from_level]
        to = op.metadata[:to_level]
        println("  $(name): $(from) → $(to)")
    end
end

println("\nPorts:")
for (name, p) in D_hjepa.ports
    println("  $(name) ($(p.direction)): $(p.ref)")
end
```

```
=== H-JEPA: 3 Abstraction Levels ===

Objects per level:
  fine: fine__Obs_fine → fine__CtxRepr_fine
  medium: medium__Obs_medium → medium__CtxRepr_medium
  coarse: coarse__Obs_coarse → coarse__CtxRepr_coarse

Abstraction morphisms:
  abstract_fine_to_medium: fine → medium
  abstract_medium_to_coarse: medium → coarse

Ports:
  input (INPUT): fine__Obs_fine
  fine_repr (OUTPUT): fine__CtxRepr_fine
  coarse_repr (OUTPUT): coarse__CtxRepr_coarse
```

Part 6: Energy-Based Cost Module

The energy function measures compatibility in representation space. The cost module decomposes into intrinsic (immutable) and trainable components:

$$C(s) = \sum_i u_i \cdot \mathrm{IC}_i(s) + \sum_j v_j \cdot \mathrm{TC}_j(s)$$

```
# Build an energy block with VICReg-style regularization
D_energy = energy_block(;
    name=:JEPAEnergy,
    energy_type=:l2,
    variance_weight=0.5,
    covariance_weight=0.1,
    collapse_strategy=VICREG,
)

cost_mods = get_cost_modules(D_energy)
cm = cost_mods[:cost]
println("=== Energy-Based Cost Module ===")
println("Intrinsic costs:")
for ic in cm.intrinsic_costs
    println("  ", ic)
end
println("\nCollapse prevention: VICReg (variance + covariance regularization)")
```

```
=== Energy-Based Cost Module ===
Intrinsic costs:
  IntrinsicCost(:prediction_cost, type=prediction, weight=1.0)
  IntrinsicCost(:variance_cost, type=variance, weight=0.5)
  IntrinsicCost(:covariance_cost, type=covariance, weight=0.1)

Collapse prevention: VICReg (variance + covariance regularization)
```

Energy Function Implementations

```
x = [1.0, 0.5, -0.3, 0.8]
y = [0.9, 0.6, -0.2, 0.7]

println("L2 energy:       ", round(energy_l2(x, y); digits=6))
```

```
println("Cosine energy:   ", round(energy_cosine(x, y); digits=6))
println("Smooth L1:       ", round(energy_smooth_l1(x, y); digits=6))

# Self-similarity should give zero energy
println("\nSelf-similarity:")
println("  L2(x, x) =     ", energy_l2(x, x))
println("  Cosine(x, x) = ", round(energy_cosine(x, x); digits=10))
```

```
L2 energy:       0.04
Cosine energy:   0.007994
Smooth L1:       0.02

Self-similarity:
  L2(x, x) =     0.0
  Cosine(x, x) = 5.1e-9
```

VICReg Regularization

```
using Random
Random.seed!(42)

# Good representations: diverse (high variance, low correlation)
Z_good = randn(4, 10)  # 4 dims, 10 samples
var_good = variance_regularization(Z_good; gamma=1.0)
cov_good = covariance_regularization(Z_good)

# Collapsed representations: all identical
Z_bad = ones(4, 10)
var_bad = variance_regularization(Z_bad; gamma=1.0)
cov_bad = covariance_regularization(Z_bad)

println("Diverse representations:")
println("  Variance penalty:   ", round(var_good; digits=4))
println("  Covariance penalty: ", round(cov_good; digits=4))
println("\nCollapsed representations:")
println("  Variance penalty:   ", round(var_bad; digits=4), " (high = BAD)")
println("  Covariance penalty: ", round(cov_bad; digits=4))
```

```
Diverse representations:
```

```
  Variance penalty:   0.4483
  Covariance penalty: 0.7101

Collapsed representations:
  Variance penalty:   3.96 (high = BAD)
  Covariance penalty: 0.0
```

Part 7: Lean 4 Proof Certificates

FunctorFlow.jl can generate Lean 4 proof certificates for JEPA diagrams, formally verifying the categorical structure:

```
D_proof = jepa_block(; name=:ProvedJEPA)
add_energy_function!(D_proof, :compat;
                     domain=[:ContextRepr, :TargetRepr], energy_type=:l2)
add_bisimulation!(D_proof, :enc_equiv;
                  coalgebra_a=:jepa_dynamics,
                  coalgebra_b=:jepa_dynamics,
                  relation=:predictor)

cert = render_jepa_certificate(D_proof)
println(cert)
```

```
-- Auto-generated by FunctorFlow.jl
namespace FunctorFlowProofs.Generated.ProvedJEPA

open FunctorFlowProofs in

def exportedDiagram : DiagramDecl := {
  name := "ProvedJEPA",
  objects := ["Observation", "Target", "ContextRepr", "TargetRepr"],
  operations := [{ name := "context_encoder", kind := OperationKind.morphism, refs := ["Observ
  ports := [{ name := "context_input", ref := "Observation", kind := "object", portType := "ob
}

def exportedArtifact : LoweringArtifact := {
  diagram := exportedDiagram,
  resolvedRefs := true,
  portsClosed := true
}
```

```
theorem exportedArtifact_checks : exportedArtifact.check = true := by native_decide

theorem exportedArtifact_sound : exportedArtifact.Sound :=
  LoweringArtifact.sound_of_check exportedArtifact_checks


-- Coalgebra declarations
def coalgebra_jepa_dynamics : CoalgebraDecl := {
  name := "jepa_dynamics",
  state := "ContextRepr",
  transition := "predictor",
  functorType := "identity"
}

-- JEPA prediction-as-obstruction theorems
/-- The JEPA prediction loss is an obstruction to commutativity
    of the encoder/predictor square. When loss = 0, the square
    commutes and the encoder is an exact coalgebra morphism. -/
theorem jepa_prediction_loss_is_obstruction :
    exportedArtifact.lossIsObstruction "prediction_loss" := by
  exact LoweringArtifact.loss_obstruction_of_check exportedArtifact_checks

/-- When the JEPA prediction loss is zero, the encoder-predictor
    path commutes with the target encoder path, making the
    encoder a coalgebra morphism (structure-preserving map). -/
theorem jepa_exact_implies_coalgebra_morphism
    (h : ∀ l ∈ exportedArtifact.losses, l.value = 0) :
    exportedArtifact.CoalgebraExact := by
  exact LoweringArtifact.coalgebra_exact_of_zero_loss h

-- Bisimulation declarations
def bisim_enc_equiv : BisimulationDecl := {
  name := "enc_equiv",
  coalgebraA := "jepa_dynamics",
  coalgebraB := "jepa_dynamics",
  relation := "predictor"
}

/-- Two coalgebras are bisimilar iff they map to the same element
    in the final coalgebra — behavioral equivalence. -/
theorem bisimilar_iff_final_coalgebra_equal
    (A B : CoalgebraDecl) (R : BisimulationDecl)
    (h : R.isBisimulation A B) :
```

```
    A.finalImage = B.finalImage :=
  CoalgebraDecl.bisim_implies_final_eq h

-- Energy function declarations
def energy_compat : EnergyDecl := {
  name := "compat",
  domain := ["ContextRepr", "TargetRepr"],
  energyType := "l2"
}

/-- Energy functions are non-negative for L2 and cosine types. -/
theorem energy_nonneg (e : EnergyDecl)
    (h : e.energyType ∈ ["l2", "cosine"]) :
    0 ≤ e.evaluate := by
  exact EnergyDecl.nonneg_of_standard h

end FunctorFlowProofs.Generated.ProvedJEPA
```

Key theorems generated: - **`jepa_prediction_loss_is_obstruction`**: The JEPA loss IS an obstruction to commutativity - **`jepa_exact_implies_coalgebra_morphism`**: Zero loss ⇒ encoder preserves structure - **`bisimilar_iff_final_coalgebra_equal`**: Bisimilar encoders are behaviorally equivalent - **`energy_nonneg`**: Energy functions are non-negative

Part 8: EMA Update (Collapse Prevention)

JEPA prevents representation collapse by using an exponential moving average (EMA) of the online encoder as the target encoder:

```
# Simulate EMA training dynamics
target_params = [Float32[0.1, 0.2, 0.3]]
online_params = [Float32[0.5, 0.6, 0.7]]

println("Before EMA:")
println("  Target: ", target_params[1])
println("  Online: ", online_params[1])

for step in 1:5
    ema_update!(target_params, online_params; decay=0.99)
end

println("\nAfter 5 EMA steps (decay=0.99):")
```

```
println("  Target: ", round.(target_params[1]; digits=4))
println("  Online: ", online_params[1], " (unchanged)")
println("\nTarget slowly tracks online → prevents collapse without negatives")
```

```
Before EMA:
  Target: Float32[0.1, 0.2, 0.3]
  Online: Float32[0.5, 0.6, 0.7]

After 5 EMA steps (decay=0.99):
  Target: Float32[0.1196, 0.2196, 0.3196]
  Online: Float32[0.5, 0.6, 0.7] (unchanged)

Target slowly tracks online → prevents collapse without negatives
```

Summary

Construction	FunctorFlow Type	Purpose
Coalgebra	`Coalgebra`	World model (state → F(state))
Coalgebra morphism	`CoalgebraMorphism`	Structure-preserving encoder
Final coalgebra	`FinalCoalgebraWitness`	Optimal representation (Lambek)
Bisimulation	`Bisimulation`	Behavioral equivalence of encoders
JEPA block	`jepa_block()`	Encoder/predictor/target triple
KAN-JEPA	`kan_jepa_block()`	Σ-attention as JEPA predictor
H-JEPA	`hjepa_block()`	Multi-scale nested coalgebras
Energy function	`EnergyFunction`	Compatibility measure in repr space
Cost module	`CostModule`	IC + TC decomposition
EMA update	`ema_update!()`	Collapse prevention

The key insight: JEPA's prediction loss is an *obstruction to diagram commutativity*. Minimizing it drives the encoder toward being an exact coalgebra morphism — a structure-preserving map between the observation world model and the latent world model.